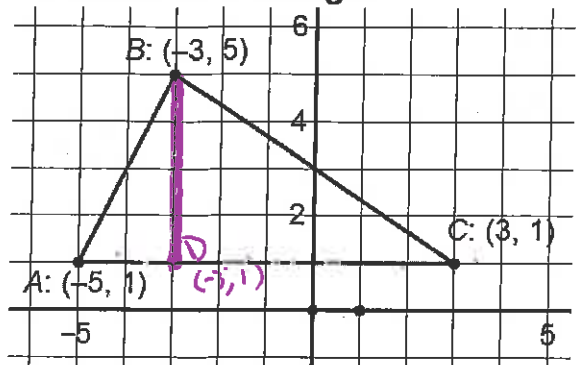


Coordinate Geometry & Triangles

Perimeter of a Triangle:



1. The perimeter of a triangle can be found by adding up the lengths of each of its sides. How can you find the length of each side of this triangle?

I can use the distance formula to find the length of each side of the \triangle .

2. Find the perimeter of this triangle.

$$\begin{aligned} AB &= \sqrt{(-3 - (-5))^2 + (5 - 1)^2} \\ &= \sqrt{(2)^2 + (4)^2} \\ &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-3 - 3)^2 + (5 - 1)^2} \\ &= \sqrt{(-6)^2 + (4)^2} \\ &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-5 - 3)^2 + (1 - 1)^2} \\ &= \sqrt{(-8)^2 + 0^2} \\ &= \sqrt{64} \end{aligned}$$

⊙ Perimeter of $\triangle ABC = \sqrt{20} + \sqrt{52} + \sqrt{64} \approx 19.68$ units.

Area of a Triangle: $A = \frac{1}{2}bh$

- b is the **base** of the triangle. It can be any one of the three sides.

- h is the **height** of the triangle. The height must be **perpendicular** to the chosen base. The height may or may not end up being one of the sides of the triangle.

3. Draw in the height of triangle ABC above. Which side of the triangle did you choose as the base? How do you know the height you drew is perpendicular to the base you chose?

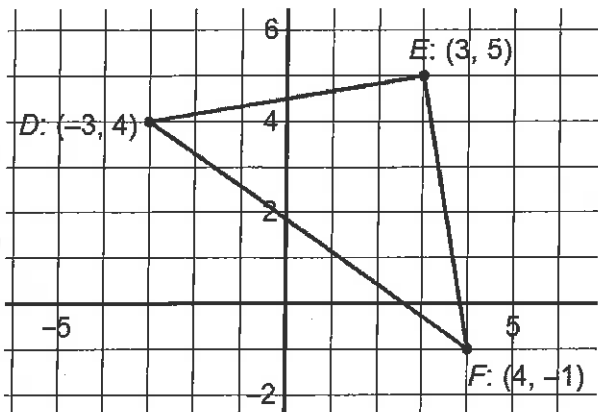
I drew Altitude \overline{BD} to Side \overline{AC} . I know: \overline{BD} is perpendicular to \overline{AC} because vertical lines and horizontal lines are always perpendicular.

4. Find the area of triangle ABC.

$$\begin{aligned} \text{height } BD &= \sqrt{(-3-3)^2 + (5-1)^2} \\ &= \sqrt{0^2 + 4^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{64} \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} b \cdot h \\ &= \frac{1}{2} (8)(4) \\ &= 16 \text{ sq. units.} \end{aligned}$$



5. Consider triangle DEF. How is finding the height of this triangle more difficult than in triangle ABC?

none of the altitudes that can be drawn are vertical or horizontal. this will make it difficult to get the altitude perpendicular to the base.

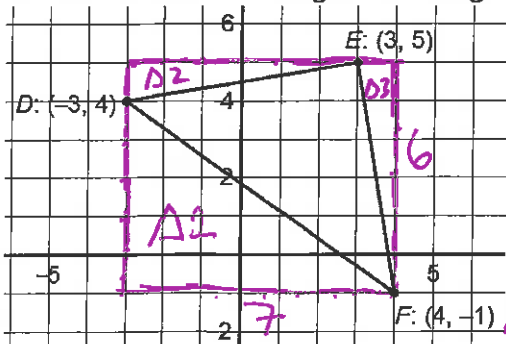
Box Method for Area:

Because the height of triangle DEF can't easily be determined, we can find the area of the triangle using the "box" method.

Steps:

1. Surround the triangle with a rectangle (or box), such that the sides are either vertical or horizontal.
2. Find the area of the box by multiplying the base times the height.
3. Subtract the areas of the smaller triangles at the corners from the area of the box.

6. Find the area of triangle DEF using the box method.



$$\text{Box Area} = 7(6) = 42 \text{ sq. units.}$$

$$\text{Area } A1 = \frac{1}{2} (7)(5) = \frac{1}{2} (35) = 17.5 \text{ sq. units.}$$

$$\text{Area } A2 = \frac{1}{2} (6)(1) = 3 \text{ sq. units}$$

$$\text{Area } A3 = \frac{1}{2} (1)(6) = 3 \text{ sq. units}$$

$$\text{Area of } \triangle DEF = 42 - (17.5 + 3 + 3) = 18.5 \text{ sq. units.}$$

Coordinate Geometry Proofs:

7. In Triangle DEF, you may have considered using \overline{DE} and \overline{FE} as the base and height respectively because they look like they might be perpendicular. Can you just **assume** they are perpendicular? Explain your reasoning.

No, we can't assume they are perpendicular. They might be close to 90° but we can't tell from the picture.

If the lines are not exactly perpendicular then that will throw off our area calculation.

8. The only way to tell for certain that \overline{DE} and \overline{FE} are perpendicular is to **prove** that they are. To "prove" means to give reasons why something is true based on evidence or facts. What evidence can you give to support an argument for \overline{DE} and \overline{FE} being perpendicular?

I could find their slopes and show the slopes are negative reciprocal values.

9. Prove $\overline{DE} \perp \overline{FE}$ by using slopes as your evidence.

$$\text{Slope } \overline{DE} = \frac{5-4}{3-(-3)} = \frac{1}{6}$$

$$\text{Slope } \overline{FE} = \frac{5-(-1)}{3-4} = \frac{6}{-1}$$

perpendicular lines have negative reciprocal slopes.
these slopes are negative reciprocal.
this proves $\overline{DE} \perp \overline{FE}$.

10. Prove that triangle DEF is an **Isosceles Triangle**.

$$\begin{aligned} DE &= \sqrt{(3-(-3))^2 + (5-4)^2} \\ &= \sqrt{6^2 + 1^2} \\ &= \sqrt{37} \end{aligned}$$

$$\begin{aligned} FE &= \sqrt{(3-4)^2 + (5-(-1))^2} \\ &= \sqrt{(-1)^2 + 6^2} \\ &= \sqrt{37} \end{aligned}$$

An isosceles triangle has 2 sides of equal length.

Since $DE = FE$, $\triangle DEF$ must be isosceles.

